General Factorial Tutorial
(Part 1 – Categoric Treatment)

Introduction – A Case Study on Battery Life

Design-Expert® software version 8 offers a "General Factorial" option on the "Factorial" tab. If you have completed the General One-Factor Tutorial (recommended), you've seen how this option handles one multilevel, categorical factor. In this two-part tutorial you will learn how to set up a design for multiple categorical factors. Part 2 shows you how to convert truly continuous factors, such as temperature, from categorical to numerical. With this you can generate response surface graphs that provide a better perspective of your system.

The experiment in this case, which comes from Montgomery's *Design and Analysis of Experiments,* seeks consistently long life in a battery that will be subjected to extremes in ambient conditions. It evaluates three materials (factor A) at three levels of temperature (factor B). Four batteries are tested at each of the nine two-factor combinations in a completely randomized design. The responses from the resulting 36 runs are shown below.

Material Type	Temperature (deg F)					
	15		70		125	
A1	130	155	34	40	20	70
	74	180	80	75	82	58
A2	150	188	136	122	25	70
	159	126	106	115	58	45
A3	138	110	174	120	96	104
	168	160	150	139	82	60

General factorial on battery (response is life in hours)

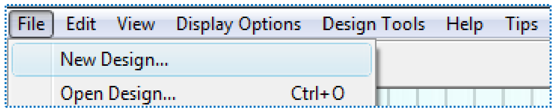
The following questions must be answered:

- How does material type and temperature affect battery life?
- Do any materials provide uniformly long life regardless of temperature?

The second question, if it can be answered in the affirmative, leads to the big payoff: a battery that will be robust to temperature variations in the field. This case study provides a good example of applying statistical DOE for robust product design.

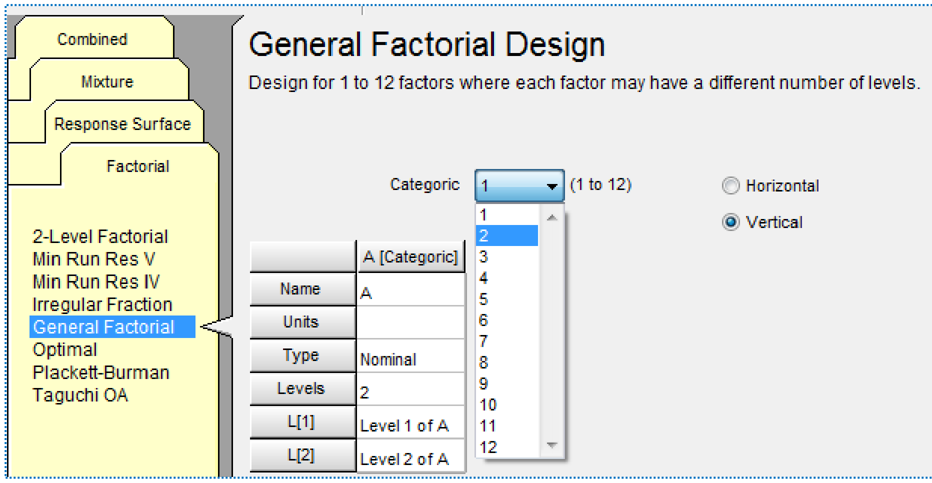
Design the Experiment

To build the design, choose **File**, **New Design** as shown below (or to save strokes, simply click the blank-sheet icon (🗋) on the toolbar).



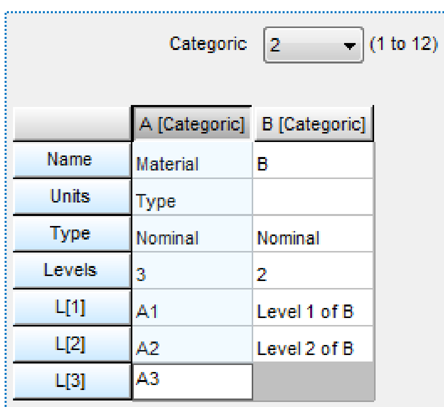
Starting a new design via menu (option: click blank-sheet icon (🗋) on the toolbar)

Then from the default **Factorial** tab, click **General Factorial**. Choose **2** as the number of factors. If you are in Horizontal entry mode, change it to **Vertical**. (Design-Expert will remember this the next time you set up a design.)



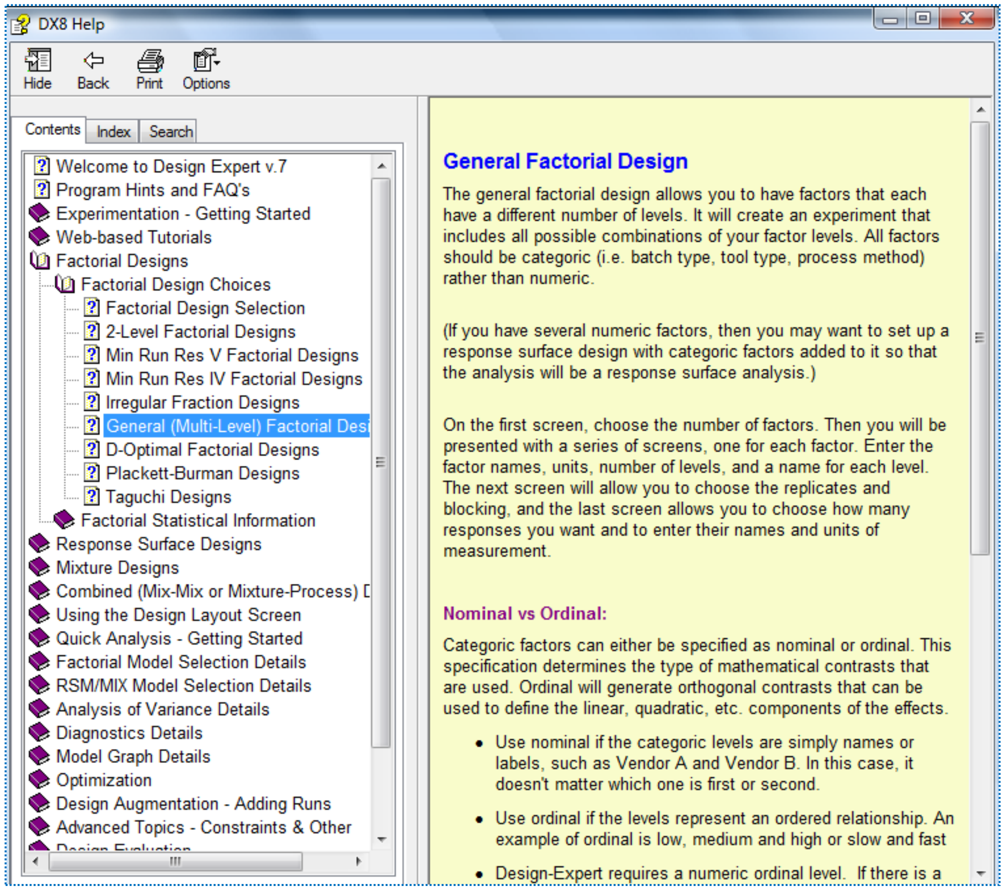
Selecting number of factors for general factorial design

Enter **Material** for factor name A (Categoric). Key in the word **Type** as your **Units**. Enter the value **3** for the number of levels. Change the treatment names to **A1**, **A2** and **A3**. Notice that **Type** in the far left column defaults to **Nominal** (named) as opposed to ordinal (ordered). This difference in the nature of factors affects how Design-Expert codes the categorical levels, which changes the model coefficients reported under ANOVA in the subsequent response analysis. Your design should now appear as that shown below.



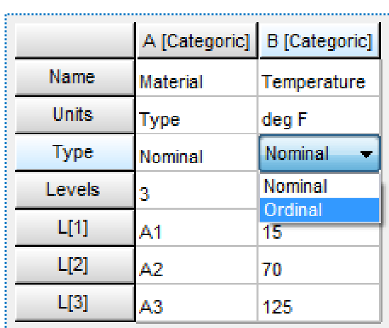
Entering material as a nominal factor

Tutorials such as this one on general factorials will quickly get you up to speed on how to use Design-Expert software, but it does not serve as a statistical primer for design and analysis of experiments. If you crave such details, Help is at your finger tips! For example, go to **Help**, **Contents** and work your way down the tree structure through the factorial branches to **General (Multi-Level) Factorial Design**. Note the details on the distinction in categoric contrasts (Nominal vs Ordinal).



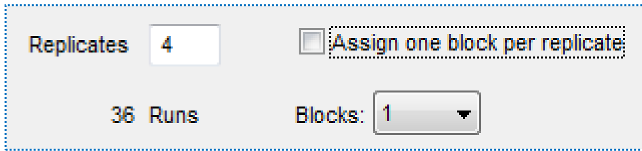
Help on general factorial design

Close Help by pressing **X** on its window. Now enter factor B data by keying in **Temperature** for factor name B (Categoric), **deg F** for units, **3** for the number of levels, and **15**, **70** and **125** for the levels. Press **Nominal**, click the arrow on the drop list, then choose **Ordinal** as shown below. This change from **Nominal** to **Ordinal** indicates that although this factor is being treated categorically (for example, due to controls offering only the three levels), temperature is really a continuous factor. Click **Continue** at the screen's far lower right.



Entering information on factor B

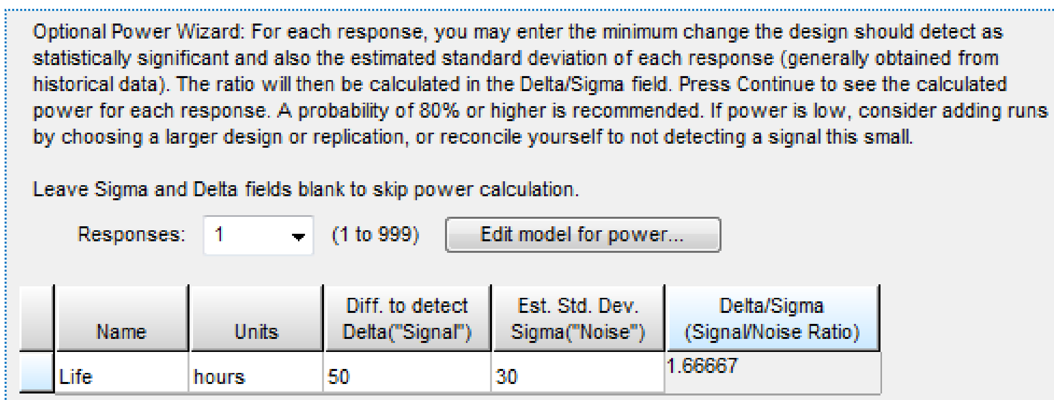
Enter **4** for replicates. The number of runs (36) won't be updated until you press the Tab key or move from the cell. Leave the blocks option alone because these experiments are completely randomized.



Entering the number of replicates

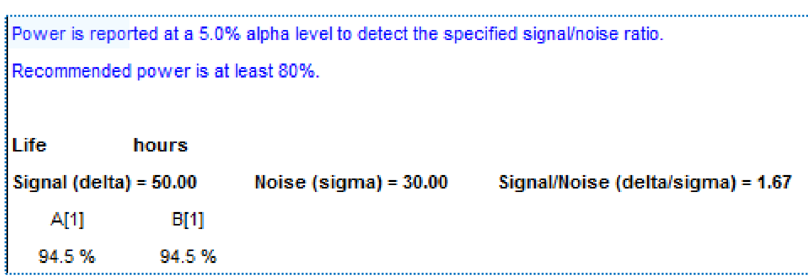
Click **Continue** to move on to the entry screen for responses. Leave the default responses at 1. Enter name as **Life** and units as **hours**.

Now we will walk you through a calculation of power – the ability of your experiment to detect meaningful differences in treatments. If you do too few runs and underpower your experiment, an important change in response (the "signal") will become obscured by normal system/test variation (the "noise"). That would be a waste of time and materials. Design-Expert makes the calculation of power easy and puts it in upfront in the design-building process so you have a chance to bolster your experiment, if necessary. Let's assume that battery life must improve by at least **50** hours to be of any interest and that quality control records produce a standard deviation of **30**. Enter these values as shown below, **Tab** (or click) out of **30,** and Design-Expert then calculates the signal to noise ratio.



Response entry screen

Press **Continue** to see the power of this design for the difference that the engineers hope to detect, at a minimum. It is calculated to be 94.5 % probability of seeing a difference (delta) as small as 50 hours. This exceeds the rule-of-thumb for power of 80 % at a minimum, thus it can be concluded that the planned design will suffice.

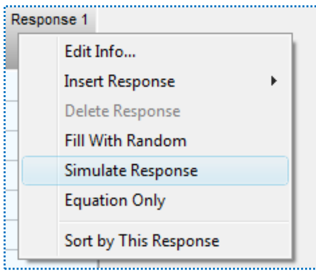


Power calculation

Click **Continue** to complete the design specification process. Design-Expert now displays the 36 runs (in random order) from the 3x3 factorial design with four replicates.

Analyze the Results

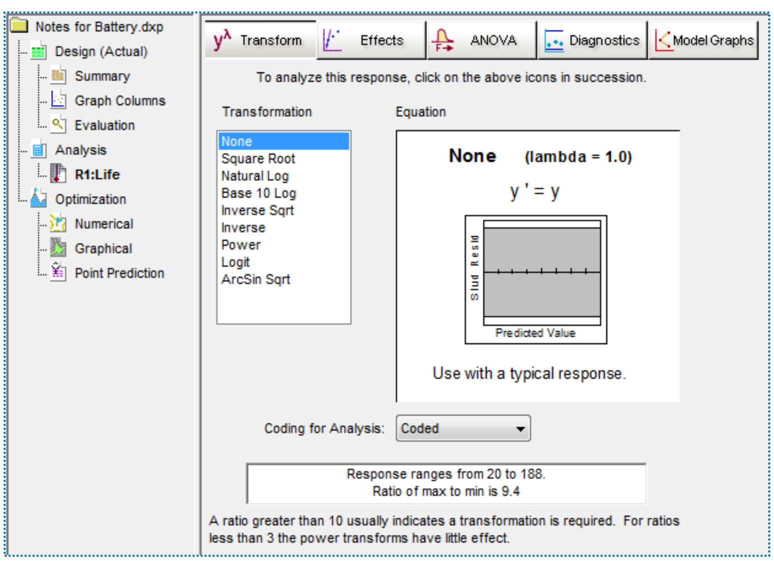
To save time, simulate the experimental results by right-clicking the response header and selecting **Simulate Response**. (A heads-up for statistics educators: You can build your own simulations via the Design Tools. Feel free to bring up the controls for this and press Help for details on using it.)



Choosing a simulation

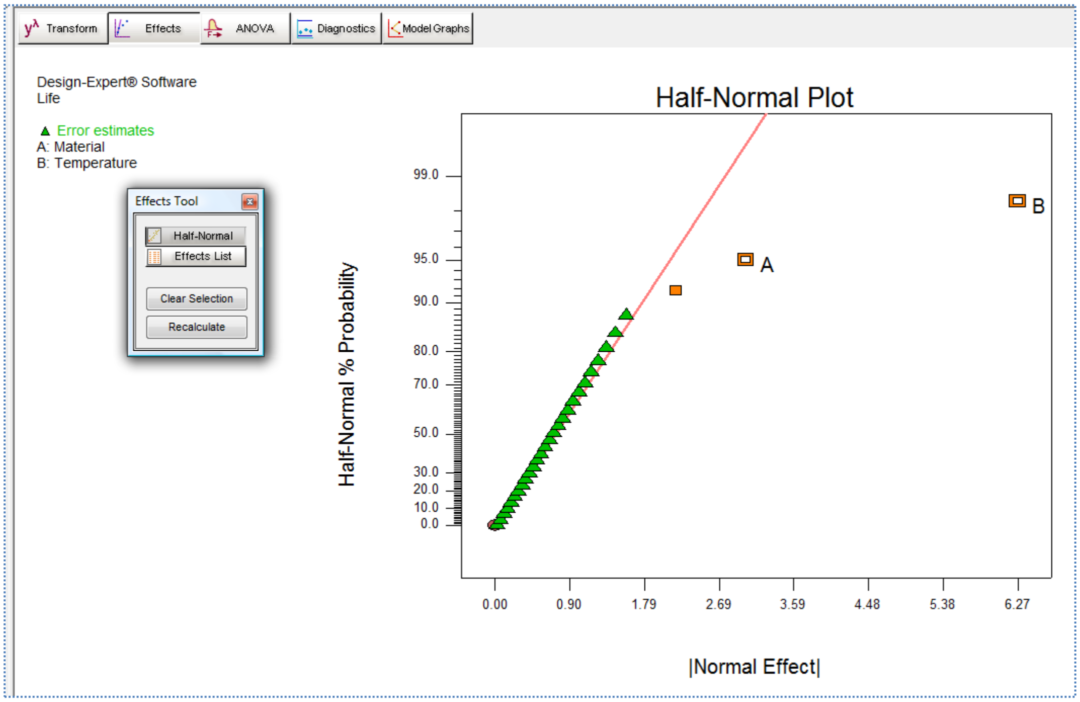
Click the file named **Battery.sim** and **Open** it. If this file does not appear, locate the data folder by finding the Design-Expert 8 data folder in the installed Stat-Ease program group under your All Programs menu. You can also download the tutorial files by going to the Help Menu – Tutorials. If you still cannot find this sim file, feel free to e-mail support@statease.com. You should now see data slowly flow in from the experiment (we added a delay in the simulator so you can read the results as they get entered by the computer – also, this makes it seem a bit harder to do the runs: Let's not make things look too easy!). This is a good time to preserve your work: Select **File** and **Save As**. Change the file name to **Battery.dxp** and **Save**.

Then under the **Analysis** branch of the program, click the node labeled **R1:Life**. You now see options for performing response transformations.



First step in the analysis – transformation options

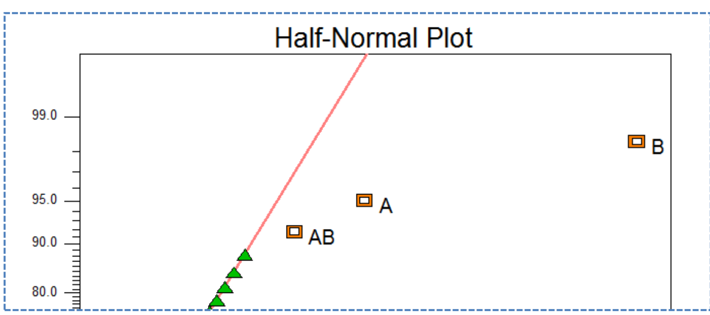
Leave the transformation at the default of "None" and go ahead and click the ***Effects*** button displayed next in the toolbar for response analysis. Design-Expert now provides an initial effect selection and displays it graphically on a specialized statistical plot called a "half-normal."



Initial effect selection

The program displays the absolute value of all effects (plotted as squares) on the bottom axis. The procedure is detailed in a presentation by Patrick Whitcomb on "Graphical Selection of Effects in General Factorials" (2007 Fall Technical Conference co-sponsored by the American Society for Quality and the American Statistical Association) – contact Stat-Ease for a copy.

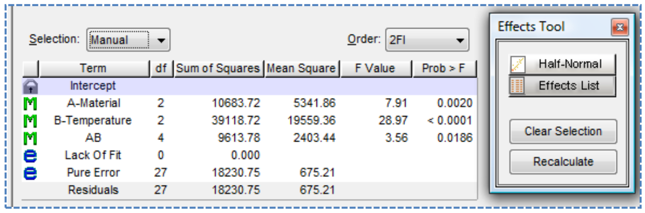
Design-Expert pre-selected two outstanding effects – the main effects of factors A and B. You can, and in this case should, modify the default effect selection. <u>Move your mouse cursor over the unlabeled square and click it</u>. (Note that this goes both ways, that is, you can deselect chosen effects with a simple mouse click.)



Another effect chosen

Interaction AB is now identified. Notice that Design-Expert adjusts the line to exclude the chosen effects. You will gain more practice on the use of half-normal plots for picking effects in the Two-Level Factorial Tutorial. It's best to now press ahead in this case.

For a numerical calculation of the effects, press the **Effect List** bar on the **Effects Tool** that you see floating on your screen (we moved it to convenient place for illustrative purposes).



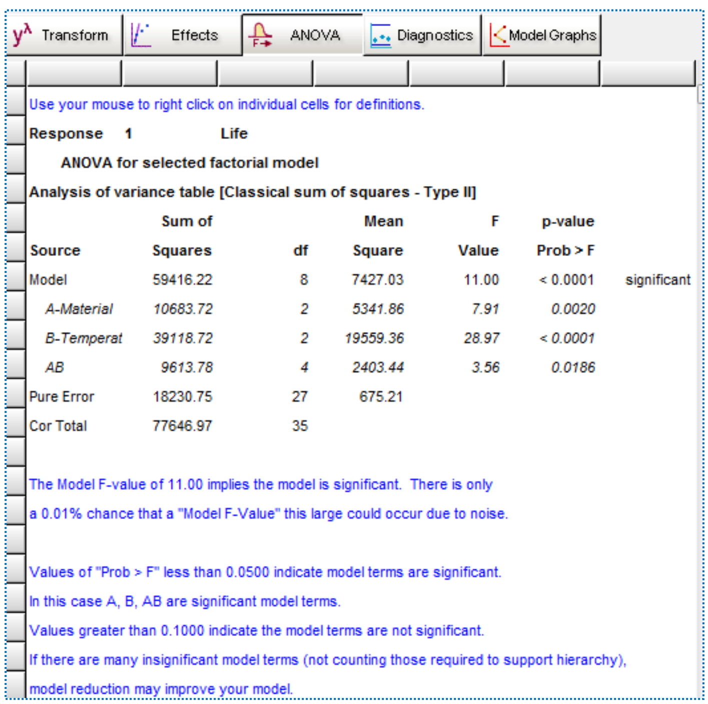
	Term	df	Sum of Squares	Mean Square	F Value	Prob > F
	Intercept					
M	A-Material	2	10683.72	5341.86	7.91	0.0020
M	B-Temperature	2	39118.72	19559.36	28.97	< 0.0001
M	AB	4	9613.78	2403.44	3.56	0.0186
e	Lack Of Fit	0	0.000			
e	Pure Error	27	18230.75	675.21		
	Residuals	27	18230.75	675.21		

Effects list

Notice the designation "M" for the selected model terms A, B and AB and the "e" next to the pure error line in this statistical spreadsheet. You may be wondering why there are so many estimates of pure error. (If not, skip ahead!) Each subgroup of 4 replications provides 3 degrees of freedom ("df") of pure error. This was done for all 9 factor combinations (3x3) which yields 27 df (= 3*9) in total for estimating pure error.

This screen provides many features for model selection, which will be covered in tutorials on response surface methods (RSM).

Click the ***ANOVA*** button to see the analysis of variance for this chosen model. If you do not see annotations in blue text as shown below, select View, Annotated ANOVA.



Use your mouse to right click on individual cells for definitions.

Response 1 Life

ANOVA for selected factorial model

Analysis of variance table [Classical sum of squares - Type II]

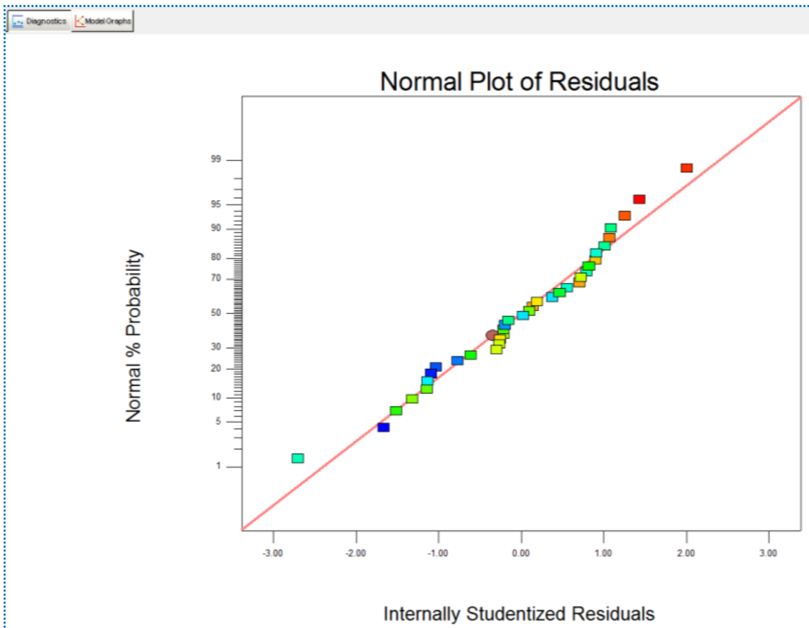
Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	59416.22	8	7427.03	11.00	< 0.0001	significant
A-Material	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*	
B-Temperat	*39118.72*	*2*	*19559.36*	*28.97*	*< 0.0001*	
AB	*9613.78*	*4*	*2403.44*	*3.56*	*0.0186*	
Pure Error	18230.75	27	675.21			
Cor Total	77646.97	35				

The Model F-value of 11.00 implies the model is significant. There is only
a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.
In this case A, B, AB are significant model terms.
Values greater than 0.1000 indicate the model terms are not significant.
If there are many insignificant model terms (not counting those required to support hierarchy),
model reduction may improve your model.

Annotated ANOVA Report

Scroll down or press bookmarks on the floating tool to see post-ANOVA statistics such as R-Squared. As you can conclude for yourself by reading the annotations, the results look good. Further down the report are details of the model based on nominal contrasts. We provide a breakdown on this in the Experiment Design Made Easy workshop. To keep this tutorial moving, it's best not to get bogged down in the mathematics of modeling categorical factors, so press ahead to the ***Diagnostics*** button and examine the residual graphs. By default you see the normal plot of residuals, which ideally fall more-or-less in line. The pattern here is not badly abnormal, so do not worry.

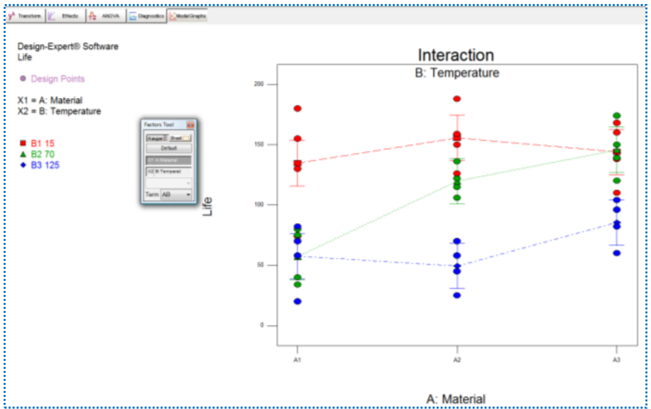


Normal plot of residuals – looks OK

The tedious, but necessary, model-fitting and statistical validation is now completed, so you are free and clear to finally assess the outcome of the experiment and decide whether any materials provide uniformly long battery-life regardless of temperature.

Present the Experimental Findings

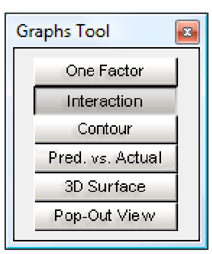
Click the ***Model Graphs*** to view the long-awaited results. Design-Expert automatically presents the AB interaction plot – identified by the Term window on the floating Factors Tool.



Default model graph – interaction plot with A on bottom (X1) axis

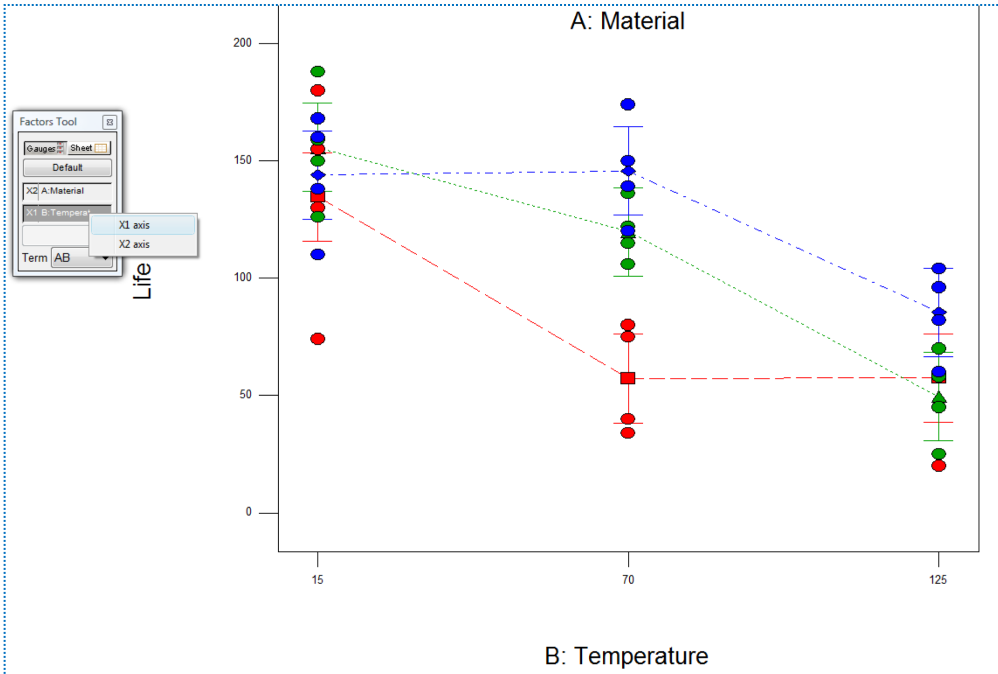
In a moment we will lead you through a series of steps to develop a compelling presentation. But first, here is a heads-up on plotting the main effects, rather than the interaction. Skip this part if you are anxious to press ahead. Otherwise, choose the One Factor plot view the View menu, or by simply pressing the appropriate button on the floating Graphs Tool. Another way to bring up a one-factor plot (the main effect of A or B, in this case) is by clicking the down-list arrow (⮟) for the Term selection on the Factors Tool. Try all these approaches if you like, but expect to be warned about presenting main effects of factors that interact. This can be very misleading. In this case it will be a mistake to look at either material or temperature effects alone, because one factor depends on the other. Let's explore this interaction further and make it as clear as possible for reporting purposes.

If you got sidetracked while exploring the program's graph options, press the Interaction plot on the Graphs Tool.



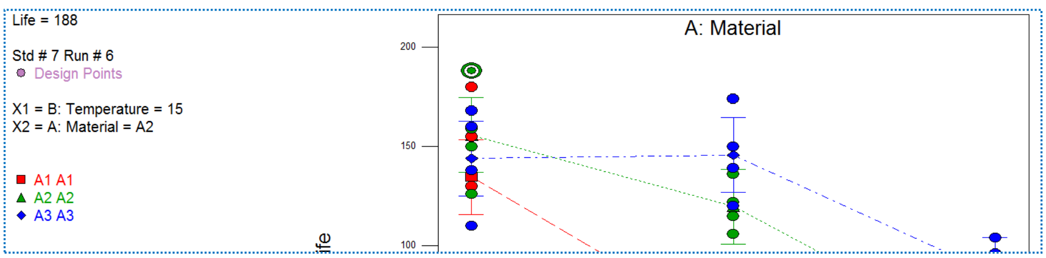
Graphs tool – interaction plot selected

Right click the **Temperature** factor on the floating **Factors Tool** and change it to the **X1 Axis**, thus producing an interaction graph with the ordinal factor displayed in a continuous manner and the nominal factor (material) laid out discretely as separate lines. This makes it easier to interpret your results.



Effect graph with temperature on bottom axis

To see how the software identifies points, click the highest one (green) at the upper left of the graph. The result is shown below.



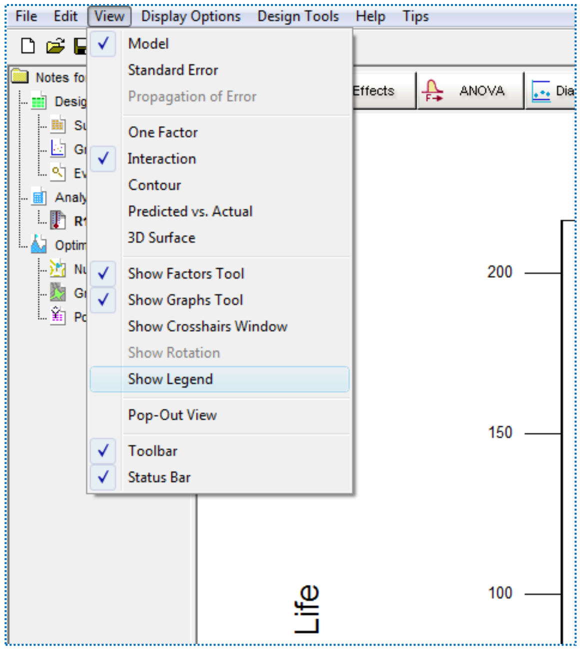
Point highlighted for identification

Note how to the left of the plot the software identifies the point by:

- the actual result (188)
- standard order number (7)
- run number (due to randomization yours may differ from that shown)
- factor levels (temperature of 15 with material A2).

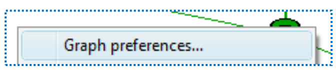
The actual results are represented by various-colored circles. You can also click on the non-circular symbols (square, triangle or diamond) to display the predicted outcome and least significant difference (LSD). Try this!

To produce a cleaner looking plot, go to **View** and <u>deselect</u> **Show Legend**.



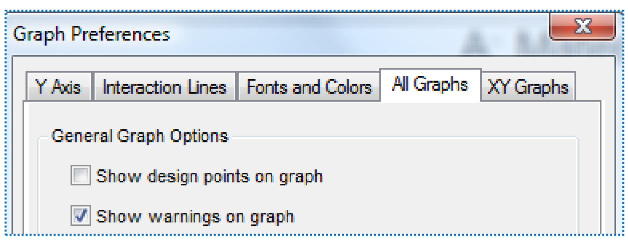
Legend turned off

Let's do some more clean-up for report purposes: <u>Right-click over the graph</u> and select **Graph Preferences**.



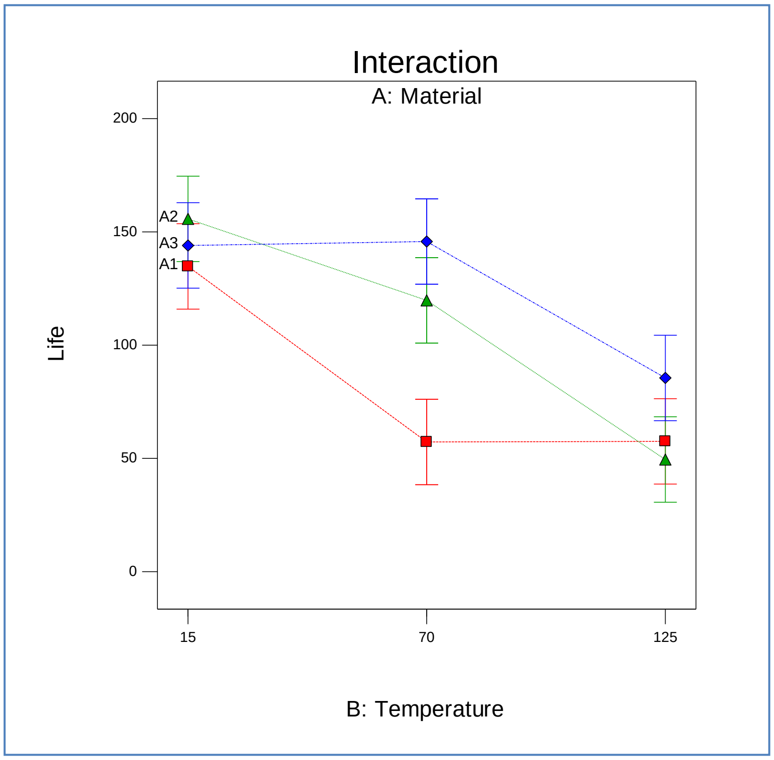
Right-click menu selection for graph preferences

Now click the **All Graphs** tab and turn off (uncheck) the **Show design points on graph** option, as shown below.



Turning off design points

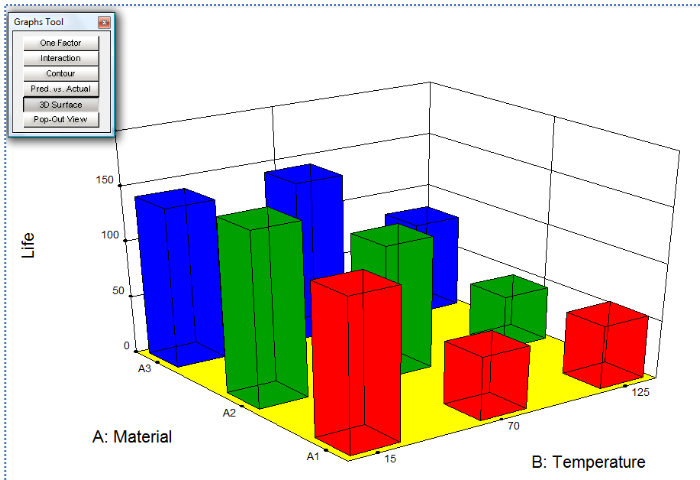
Press **OK**. This is an optional sidetrack on this tutorial: To have your graph look like that shown below for reporting purposes, do the following: Edit, Copy from Design-Expert, then Edit, Paste into Microsoft Word.



Clean-looking interaction graph

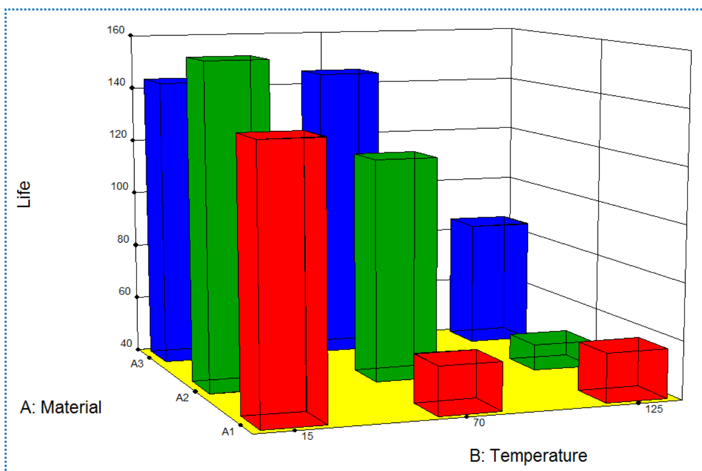
From this graph you can see that all three materials work very well at the low temperature (15 degrees). Based on the overlapping LSD bars, it would be fair to say that no material stands out at this low temperature end of the scale. However, the A1 material clearly falls off at the 70 degree temperature, which would be encountered most often, so it must be rejected. None of the materials perform very well at the highest temperature (125 degrees), but the upper end of the LSD bar for A2 barely overlaps the bottom end of the LSD bar for A3. Therefore, with respect to temperature sensitivity, material A3 may be the most robust material for making batteries.

Finally, if you do have an opportunity to present graphics in color, here's a dazzling new and easy way to display general factorial effects with Design-Expert: Click **3D Surface** on the floating **Graphs Tool**.



3D surface plot

Now place your mouse cursor on the graph – notice that it changes to a hand (✋). <u>While pressing the left mouse button</u>, spin the graph so the temperature axis is at the bottom. (Alternatively, to match our graph most precisely, select View, Show Rotation and enter coordinates of h (horizontal) 20 and v (vertical) 80.)



3D surface plot – rotated slightly for a better view

The 3D view presents a different perspective of the general factorial effects – more on a macro level of the overall experimental landscape. Now the inferiority of material A1 (red bars) becomes obvious: The other two materials tower over it at the mid-temperature of 70 degrees F. Clearly the next step is to eliminate material A1 from contention and perhaps do some further investigations on A2 and A3.